SAT Solvers in the Context of Cryptography

v2.0

Presentation at Montpellier

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Outline

Motivations and goals

Motivations

- Cryptographic primitives may be broken using SAT solving tools
- Analysis of cryptographic primitives is possible using SAT solvers

Goals

- Show why SAT solvers work well to analyse and/or break cryptographic primitives
- Draw attention to the drawbacks a and how to overcome them

What is a SAT solver

Solves a problem in CNF

CNF is an "and of or-s"

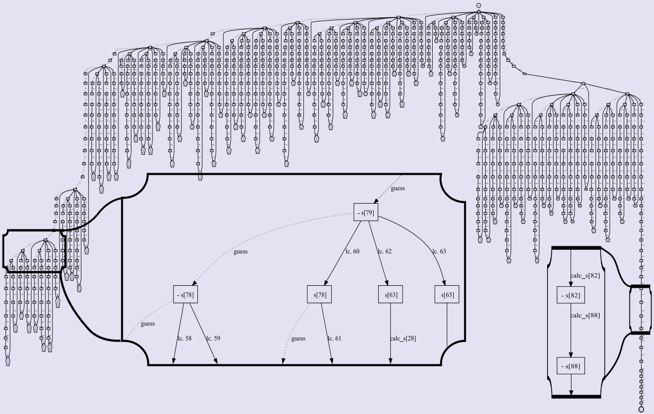
$$(x_1 \vee \neg x_3) \quad \wedge \quad (\neg x_2 \vee x_3) \quad \wedge \quad (x_1 \vee x_2)$$

Uses DPLL(φ) algorithm

1. If formula φ is trivial, return SAT/UNSAT
2. ret = DPLL(φ with $v \leftarrow$ `true`)
3. if ret == SAT, return SAT
4. ret = DPLL(φ with $v \leftarrow$ `false`)
5. if ret == SAT, return SAT
6. return UNSAT

Search tree

Example search tree



Visualised

- Guesses
- Propagations
- Generated learnt clauses
- Clause group causing the propagation

Calculated stats

- Average depth
- Most conflicted clauses
- No. of guess/branch
- Most guessed vars
- Most propagated vars

SAT solver internals

Conflict clauses

- Generated when current assignment doesn't satisfy a clause
- Collection of information leading to conflict
- Used to avoid similar wrong parts of the tree next time

Most important parts

- Lazy data structures
- Learning (and forgetting)
- How to pick a variable
- When to restart

Cryptographic problems

Stream ciphers: Grain, HiTag2

- Generates pseudorandom keystream given public IV and secret key
- Step-by-step iteration is easy to describe in ANF
- ANF is relatively easy to convert to CNF

Block ciphers: DES, AES

- Encodes a plaintext to a chipertext given a secret key
- Can have relatively difficult internal parts e.g. S-box
- May be difficult to model in CNF

Hash functions: SHA-1

- Generates one-way, (second)preimage-resistant fingerprint of text
- Usually has relatively difficult internal parts e.g. circular left-shift
- Difficult to model in CNF

Outline

Advantages of SAT solvers in the context of cryptography

Brute force

- Try every setting of the unknowns
- Apply every setting to every equation
- If doesn't work, start from the beginning

SAT solvers

- Learnt clauses
  - Act as memory
  - Trim the search tree
- Lazy data structures
  - Fast partial back-tracking
  - Keep partially computed values in memory
- Variable activity heuristics
  - Find good points of entry
  - E.g. key bits, shift register states, etc.

Learnt clauses

Memory model

- When guesses were wrong $\Rightarrow$ *conflict*
- At conflict record how we came here $\Rightarrow$ *learnt clause*
- Learnt clauses act as memory

Learnt clause erasure strategy

- Learnt clause is active in new conflict $\Rightarrow$ *activity* $\uparrow$
- Other clauses' activity $\downarrow$
- Low-activity clauses are periodically *erased*

In the context of cryptography — demonstration

- `./cryptominisat --stats hitag2-shifted-31.cnf`
- `./cryptominisat --stats grain-shift-60.cnf`

Lazy data structures

Watchlists

- Only act upon a clause when we have to
- When all literals are assigned `false` except for one → assign free literal to `true`:

$$v1 \vee v2 \vee v3$$

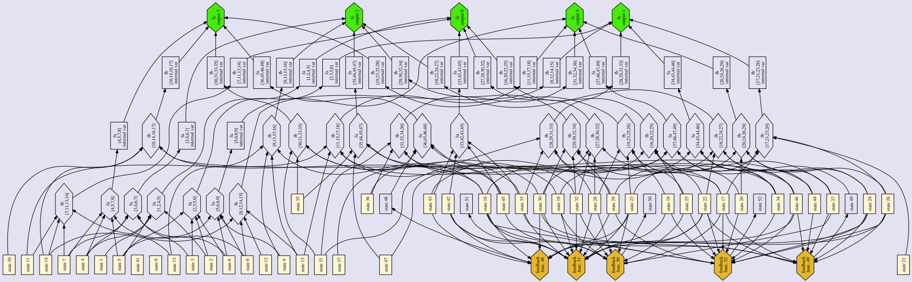
$$v1 = \texttt{false}, v2 = \texttt{false} \quad \longrightarrow \quad v3 = \texttt{true}$$

Internal variables

- It is possible to describe complex functions without internal variables using Karnaugh maps
- But it *slows down* the solving
- Many think they are a necessary evil. They in fact *help*
- They let the solver go back to a point in the search tree without the need to re-compute values

Lazy data structures

Example stream cipher

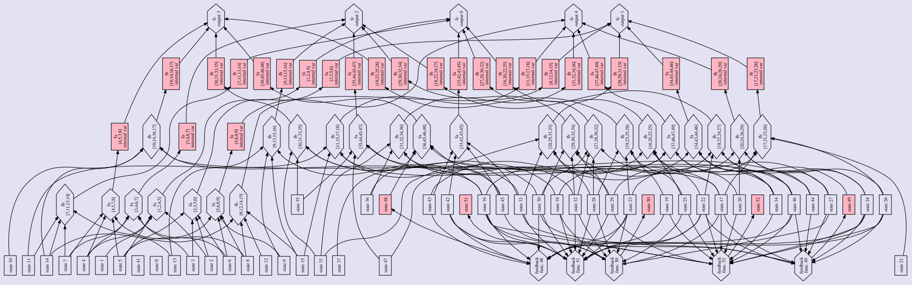


Explanation

- Hexagons: Filter and feedback functions
- Boxes: Variables (state and internal)
- Green: Final filter functions
- Yellow: Initial state
- Red: Feedback functions

Lazy data structures

Example stream cipher

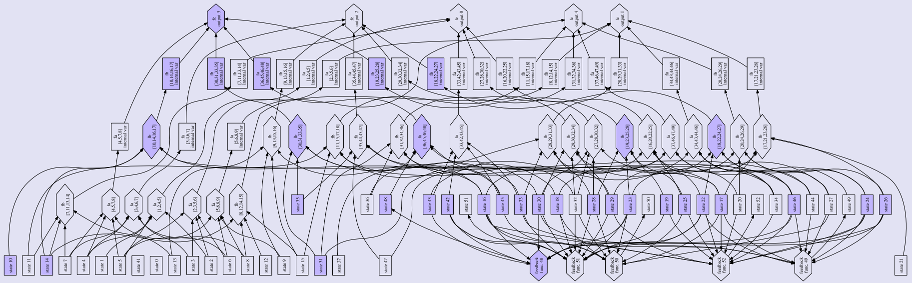


Explanation

- Hexagons: Filter and feedback functions
- Boxes: Variables (state and internal)
- Red: Internal variables

Lazy data structures

Example stream cipher



Explanation

- Hexagons: Filter and feedback functions
- Boxes: Variables (state and internal)
- Blue: Dependency of 4th output bit

Variable activity heuristics

Goal

- Make search tree balanced
- Divide the problem *equally*: $v =$ `true`, `false`
- Otherwise: unbalanced tree, search depth becomes huge
- Example:
  - Unbalanced tree: search depth = 1'000
  - Balanced tree: search depth = 100

Searching for a good branch

- Variable appears often in conflict $\Rightarrow$ activity $\uparrow$
- All other variables' activity $\downarrow$
- Most active variables branched first

Variable activity heuristics in the context of cryptography

Variable activity in crypto-problems

- Stream cipher with initialisation: branches on key bits
- Stream cipher without initialisation: branches on shift register state
- Algebraic side-channel attack: branches on internal variables of the side-channel information round

Demonstration

- HiTag2 — with initialisation
- Grain, HiTag2 — without initialisation, shifted 31,60 resp.:

- ASCA for PRESENT — not available

Outline

Disadvantages of SAT solvers in the context of cryptography

Problem structure lost

- CNFs is information-short
- Functions: Filter function? Bit-shift?
- Data: Side-channel information? Observed ciphertext?

Probabilities difficult to handle

- All clauses must be true
- How could we model $P(\text{information is correct}) = 0.4$?

Problem structure is lost

ANF vs. CNF

- Cannot find out that, e.g.

$$v1 \oplus v2 \oplus v3 = \texttt{true}$$
$$v1 \oplus v2 \oplus v4 = \texttt{true}$$
$$\therefore v3 = v4$$

- Sub-problems may be hard: e.g. Gauss elim.
- Result: simple problems become difficult

Internal variables — what do they represent?

- monomials, long xors $\Rightarrow$ too many vars
- Could disorientate variable activities
- Trivial problems may take hours to solve

Probabilities difficult to handle

Adding statistical information

- Clauses cannot have probabilities
- E.g: $P(v10 \lor v11 \lor v12 = \texttt{true}) = 0.4$
- Add multiple informations of low probability:

$$\mathbf{v1} \lor \mathbf{v2} \lor \mathbf{v3}$$

$$\begin{aligned} \text{where } v1 &\longleftrightarrow v10 \lor v11 \lor v12 \\ \text{and } v2 &\longleftrightarrow v20 \lor v21 \lor v22 \\ &\ldots \end{aligned}$$

Probabilistic information may lead to problems

- With (small) probability 0.6^3 : $v1 \lor v2 \lor v3 = \texttt{false}$
- Leads to UNSAT — need to re-start search
- Statistical information difficult to model

Outline

Conclusions

Concluding remarks

- SAT is effective at analysing cryptographic problems
- SAT can break simple cryptographic routines
- Complex ciphers: careful translation is needed

Future work

- Recover information from CNF
- Add information to the CNF
  e.g. variable categories: key, ciphertext, side-channel info
- Handle probabilistic information

Thank you for your time

Any questions?