SAT Solvers in the Context of Cryptography

Presentation at Rennes

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Motivations and goals

Motivations

- Cryptographic primitives could possibly be broken using automated SAT solving tools
- If not, then analysis of cryptographic primitives might be possible using SAT solvers

Goals

- Show why SAT solvers work so well to analyse and/or break cryptographic primitives
- Draw attention to the drawbacks and bottlenecks and how they could be overcome

What is a SAT solver

Solves a problem in CNF

CNF is an "and of or-s"

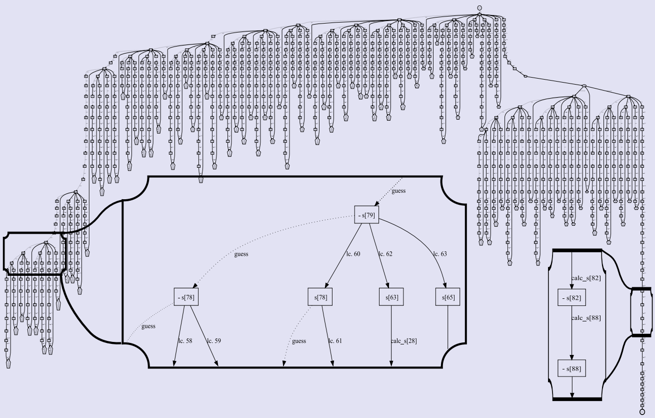
$$\neg x_1 \vee \neg x_3 \qquad \neg x_2 \vee x_3 \qquad x_1 \vee x_2$$

Uses DPLL(φ) algorithm

1. If formula φ is trivial, return SAT/UNSAT
2. Picks a variable v to branch on
3. $v \leftarrow$ value
4. Simplifies formula to φ' and calls DPLL(φ')
5. if SAT, output SAT
6. if UNSAT, $v \leftarrow$ opposite value
7. Simplifies formula to φ'' and calls DPLL(φ'')
8. if SAT, output SAT
9. if UNSAT, output UNSAT

Search tree

Example search tree



Visualised

- Guesses
- Propagations
- Generated learnt clauses
- Clause group causing the propagation

Calculated stats

- Average depth
- Most conflicted clauses
- No. of guess/branch
- Most guessed vars
- Most propagated vars

SAT solver internals

Conflict clauses

- Generated when current assignment doesn't satisfy a clause
- Collection of information leading to UNSAT
- Used to avoid similar wrong parts of the tree next time

Most important parts

- Lazy data structures
- Learning (and forgetting)
- How to pick a variable

Cryptographic problems

Stream ciphers

- Generates pseudorandom keystream given public IV and secret key
- Step-by-step iteration is easy to describe in ANF
- ANF is relatively easy to convert to CNF

Block ciphers

- Encodes a plaintext to a chipertext given a secret key
- Can have relatively difficult internal parts e.g. S-box
- May be difficult to model in CNF

Hash functions

- Generates one-way, (second)preimage-resistant fingerprint of text
- Usually has relatively difficult internal parts e.g. circular left-shift
- Difficult to model in CNF

Outline

Advantages of SAT solvers in the context of cryptography

Lazy data structures

- Fast back-tracking
- Keep partially computed values in memory

Learnt clauses

- Trim the search tree
- Act as memory

Variable activity heuristics

- Search and find good points of entry
- E.g. key bits, shift register states, etc.

Lazy data structures

Watchlists

- Only act upon a clause when we have to
- When all literals are assigned `false` except for one $\rightarrow$ assign free literal to `true`:

$$v1 \lor v2 \lor v3$$

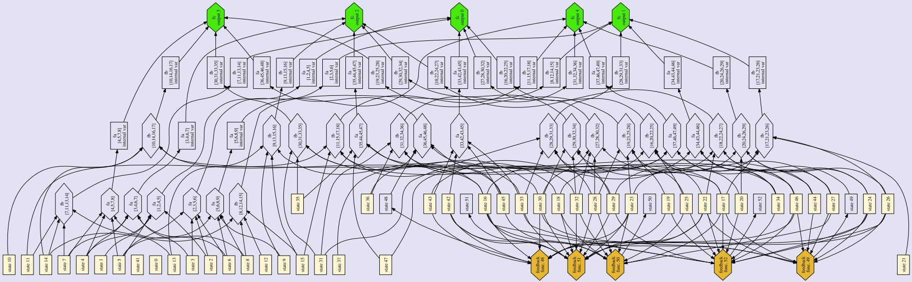
$$v1 = \texttt{false}, v2 = \texttt{false} \quad \longrightarrow \quad v3 = \texttt{true}$$

Internal variables

- It is possible to describe complex functions without internal variables using Karnaugh maps
- But it *slows down* the solving
- Many think they are a necessary evil. They in fact *help*
- They let the solver go back to a point in the search tree without the need to re-compute values

Lazy data structures

Example stream cipher

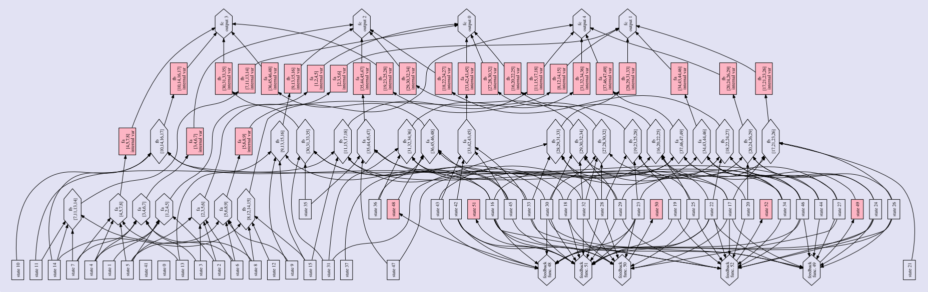


Explanation

- Hexagons: Filter and feedback functions
- Boxes: Variables (state and internal)
- Green: Final filter functions
- Yellow: Initial state
- Red: Feedback functions

Lazy data structures

Example stream cipher

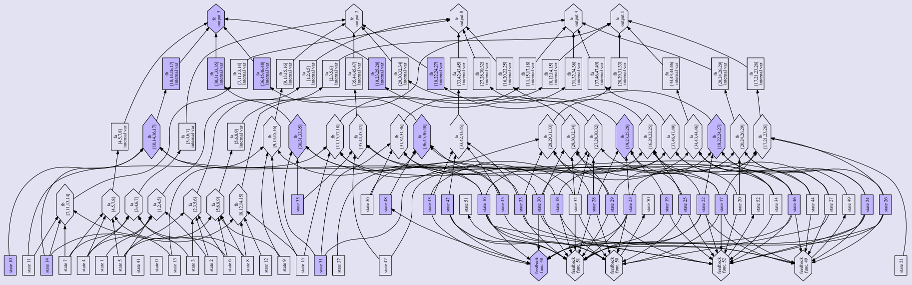


Explanation

- Hexagons: Filter and feedback functions
- Boxes: Variables (state and internal)
- Red: Internal variables

Lazy data structures

Example stream cipher



Explanation

- Hexagons: Filter and feedback functions
- Boxes: Variables (state and internal)
- Blue: Dependency of 4th output bit

Learnt clauses

Memory model

- Learnt clauses record each conflict (i.e. no solution is in that part of the search tree)
- They act as memory — but too much memory makes it hard to search effectively
- Trade-off: some learnt clauses are erased periodically

Learnt clause erasure strategy

- If a learnt clause is active part of a new conflict, its activity is increased
- All other clauses' activity is decreased
- So, learnt clauses that actively help to trim the search tree are preserved

In the context of cryptography — demonstration

```
./cryptominisat --stats --grouping hitag2.cnf | less
```

Variable activity heuristics

Searching for a good branch

- Variables are initially randomly branched on
- Variables that appear in during conflict have their activity increased
- All other variables' activity is decreased
- Most active variables are branched first

Effect of heuristic

- Most difficult variables branched on first
- Difficult = its value affects a lot of other variables
- It divides the problem *equally* into two parts: $v =$ `true` and `false`
- If less important variable is picked, the problem is not equally divided: unbalanced tree, search depth becomes huge

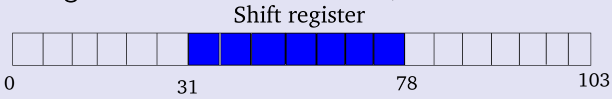
Variable activity heuristics in the context of cryptography

Variable activity in crypto-problems

- Stream cipher with initialisation: branches on key bits
- Stream cipher without initialisation: branches on shift register state
- Algebraic side-channel attack: branches on internal variables of the side-channel information round

Demonstration

- Grain — with initialisation
- HiTag2 — without initialisation, shifted 31:



- ASCA for PRESENT — not available

Outline

Disadvantages of SAT solvers in the context of cryptography

Problem structure lost

- CNFs is information-short
- Functions: Filter function? Bit-shift?
- Data: Side-channel information? Observed ciphertext?

Probabilities difficult to handle

- All clauses must be true
- How could we model $P(\text{information is correct}) = 0.4$?

Problem structure is lost

ANF vs. CNF

- Cannot find out that, e.g.

$$v1 \oplus v2 \oplus v3 = \texttt{true}$$
$$v1 \oplus v2 \oplus v4 = \texttt{true}$$
$$\therefore v3 = v4$$

- Sub-problems of crypto may be hard for SAT: e.g. Gauss elim.
- Result: information is lost, can make simple problems very difficult

Internal variables — what do they represent?

- May try to branch on variables introduced to cut up long XOR-s
- Too many monomials may lead to too many variables — could disorientate variable activity heuristics
- Result: In extreme cases, trivial problems take hours to solve

Probabilities difficult to handle

Adding statistical information

- Clauses cannot have probabilities associated with them
- Example: $P(v10 \lor v11 \lor v12 = \texttt{true}) = 0.4$. How can this be modelled?
- Solution: add multiple informations of low probability in one clause:

$$\mathbf{v1} \lor \mathbf{v2} \lor \mathbf{v3}$$

$$\begin{aligned} \text{where } v1 &\longleftrightarrow v10 \lor v11 \lor v12 \\ \text{and } v2 &\longleftrightarrow v20 \lor v21 \lor v22 \\ &\ldots \end{aligned}$$

Probabilistic information may lead to problems

- With (small) probability 0.6^3 : $v1 \lor v2 \lor v3 = \texttt{false}$
- Leads to UNSAT — need to re-start seach
- So, statistical information is difficult to model

Outline

Conclusions

Concluding remarks

- SAT solvers are an effective way to analyse cryptographic problems
- Can be used to break simple cryptographic routines automatically
- But for complex ciphers, careful translation is needed

Future work

- Recover information from CNF
  e.g. discover and effectively use XOR functions
- Add information to the CNF:
  e.g. clause categories: key, ciphertext, side-channel info
- Handle probabilistic information

Thank you for your time

Any questions?