Limits of SAT Solvers in Cryptography

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PRESENTATION AT CASED

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Story Line

1. Introduction to SAT Solvers and Cryptography
2. Advantages of SAT in Crypto
3. Limitations of SAT in Crypto

What is a SAT Solver

Solves a problem in CNF

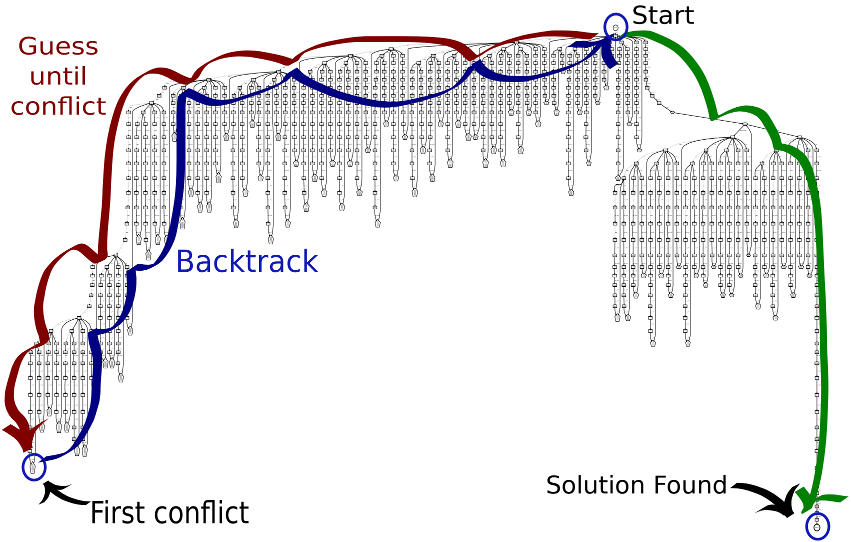
CNF is an "`and` of `or`-s"

$$\neg x_1 \vee \neg x_3 \qquad \neg x_2 \vee x_3 \qquad x_1 \vee x_2$$

Uses DPLL(φ) algorithm

1. If φ trivial, return SAT/UNSAT
2. call DPLL($v \leftarrow$ value)
3. if SAT, output solution
4. if UNSAT, call DPLL($v \leftarrow$ opposite value)
5. if SAT, output solution
6. return UNSAT

Example Search Tree



SAT Solver Internals

- Lazy data structures: watchlists, fast backtracking
- Learning (and forgetting): what to learnt and what to forget?
- Picking variables: which ones to branch on, and in what order?
- Restarts: when to restart, how far to restart

Cryptographic Problems

Stream ciphers

- Generates pseudorandom keystream given public IV and secret key
- Step-by-step iteration easy to describe in ANF
- Easy to model in CNF

Block ciphers

- Encodes a plaintext to a chipertext given a secret key
- Relatively difficult internal parts e.g. S-box
- May be difficult to model in CNF

Hash functions

- Generates one-way, (second)preimage-resistant fingerprint
- Usually has difficult internal parts e.g. adder
- Difficult to model in CNF

Advantages of SAT in Crypto

- Find good points of entry: picking variables
- Partially evaluate the function: lazy data structures
- Effectively store explored search space: learnt clauses

Limitations of SAT in Crypto

Structure lost

- CNFs is "plain" – adders, multipliers not evident
- Higher-level reasoning is very difficult
- Cannot find out that, e.g.
  $v1 \oplus v2 \oplus v3 = \texttt{true}, v1 \oplus v2 \oplus v4 = \texttt{true}$
  $\therefore v3 = v4$
- Gauss elim. needs exponential resolution operations

Probabilities difficult to handle

- All clauses must be true
- Example: $P(v10 \vee v11 \vee v12 = \texttt{true}) = 0.4$. How to model?
- Introduce indicator variable, make it depend on multiple low-probability events

One-to-one Translation has Limited Potential

Past

- One-to-one translation has been tried on many crypto-primitives
- With varying sophistication levels
- Disappointing results on strong primitives (e.g. SHA1, AES, MD5)

Future

- Don't model the algorithm one-to-one
- Model a particular aspect of it, e.g. differential path
- Challenges: what to model, how to model, how to interpret results

Conclusions

Concluding remarks

- SAT solvers can be effective on some crypto problems
- Can break simple cryptographic routines automatically
- But it's far from plug-and-play for complex crypto-primitives

Future work

- Make the plug-and-play experience better for simple problems
- Find crypto-primitive properties to model that could lead to attacks
- Refine properties modelled, refine modelling techniques

Thank you for your time

Any questions?