Algorithms Transcending the SAT-Symmetry Interface

Markus Anders Mate Soos Pascal Schweitzer

SAT 2023

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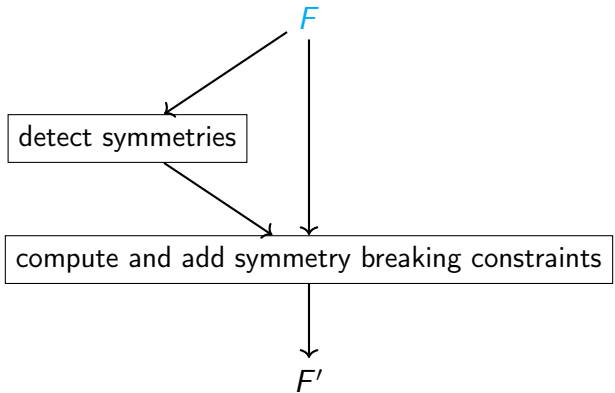
- a disjunction $C \in F$ is called a *clause*
- an element $l \in C$ is called a *literal*

Symmetry Breaking



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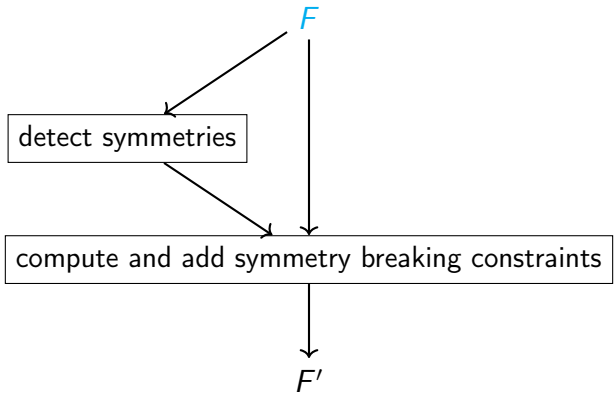

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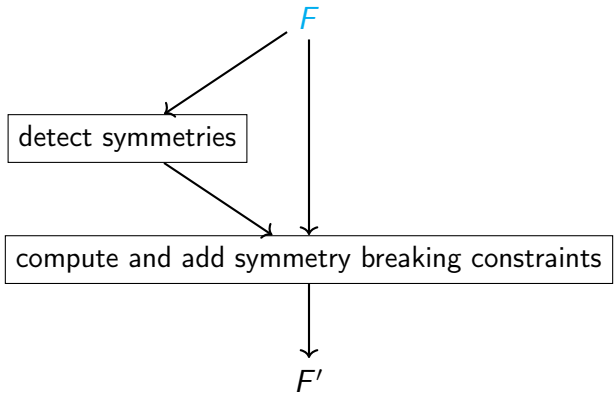

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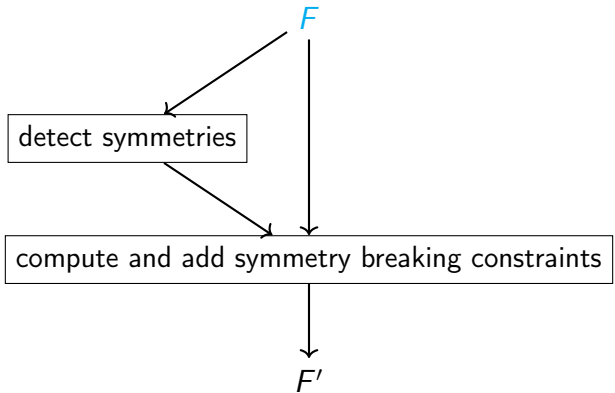

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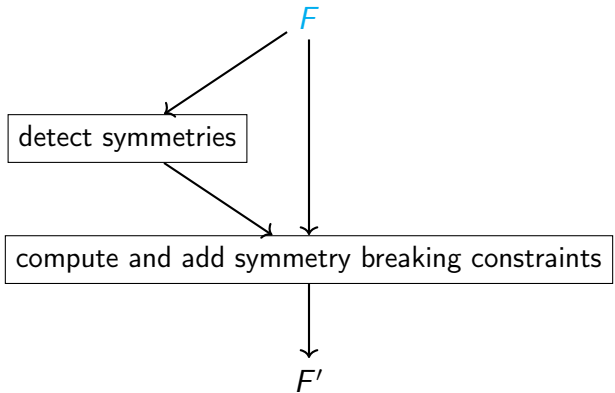

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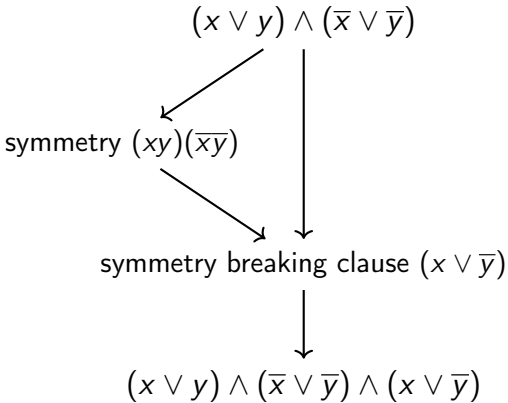
- overhead is an issue

Symmetry Breaking: Example



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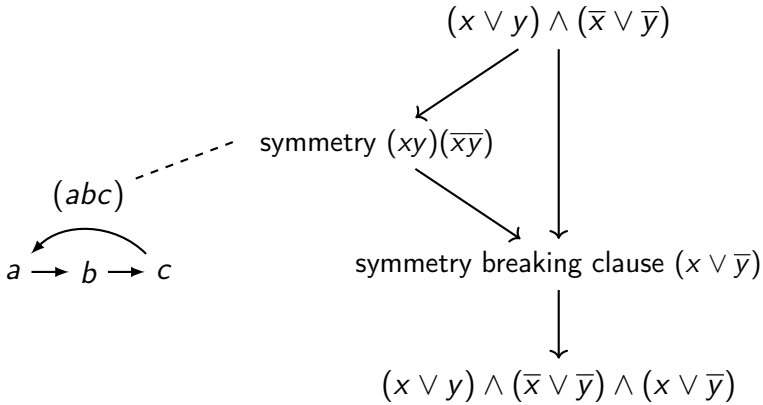

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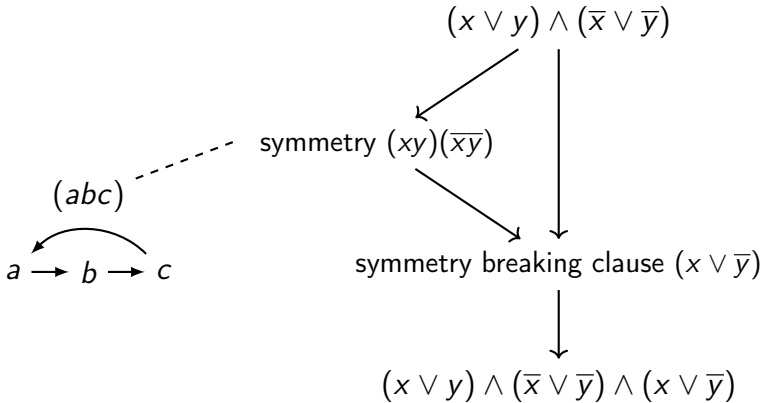

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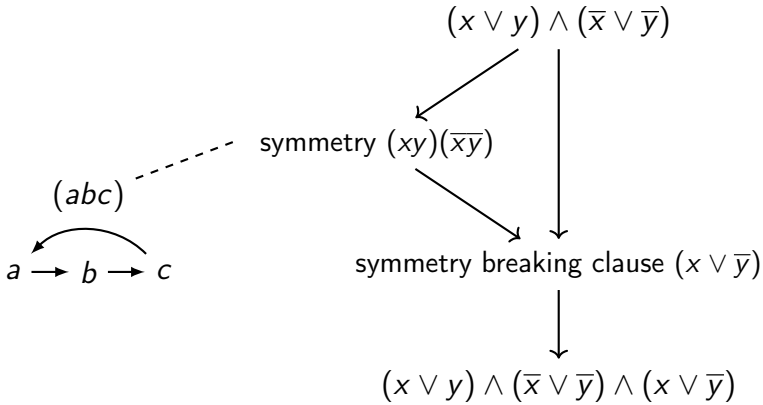

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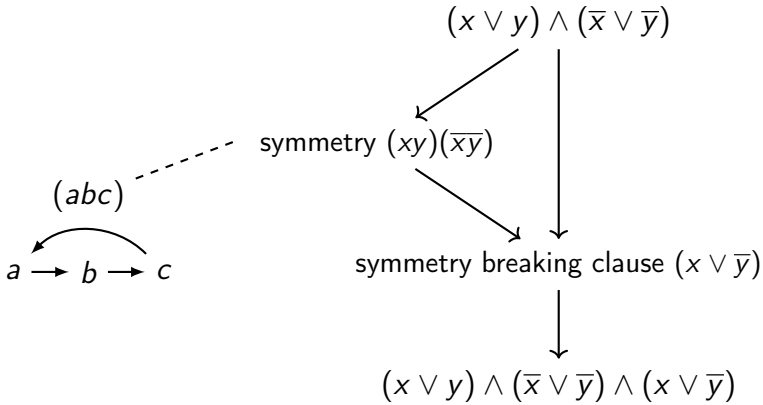

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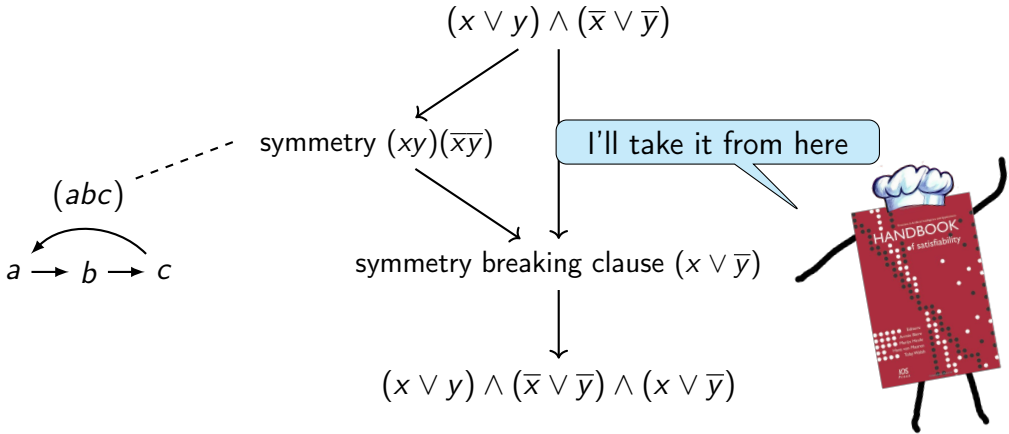

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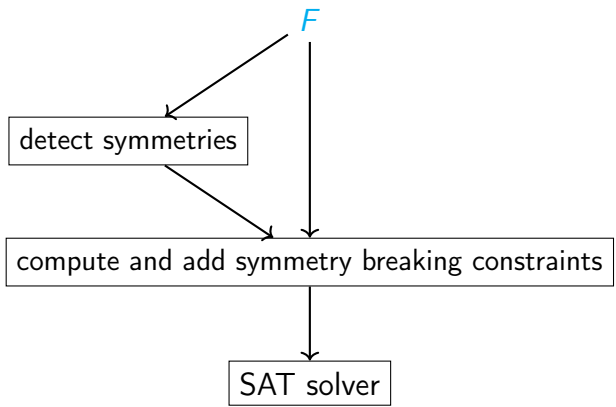

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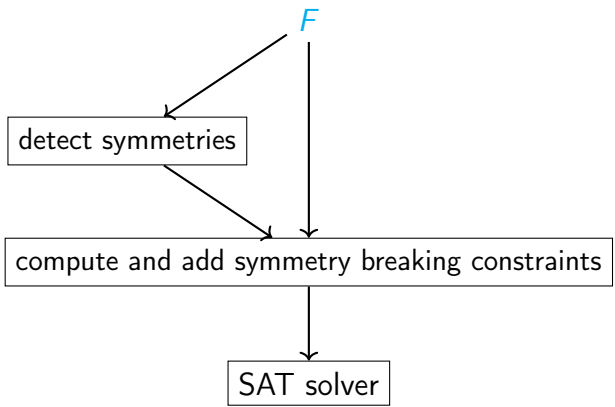
Step 2:
exploitation



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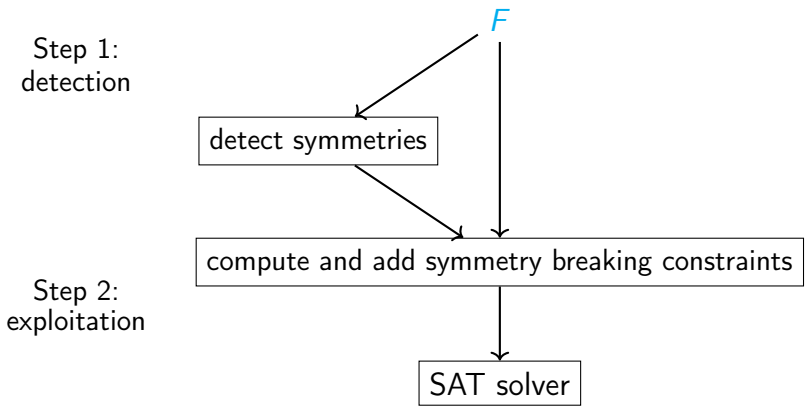

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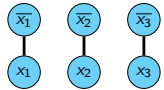

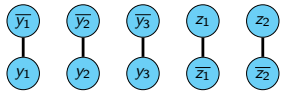

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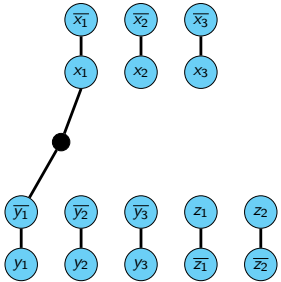

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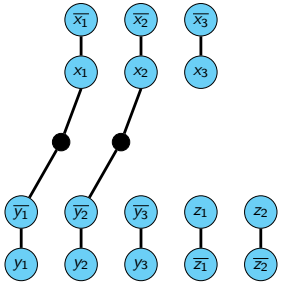

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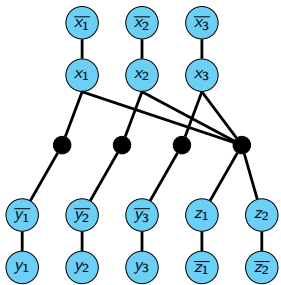

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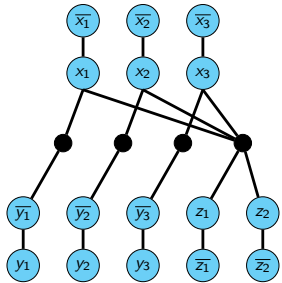

- symmetries of graph are symmetries of formula (and vice versa)

By the way... what are symmetries again? (CNF)

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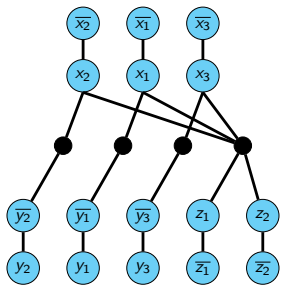
By the way... what are symmetries again? (Graphs)

- bijection of vertices $\varphi : V \to V$ with $\varphi(G) = (\varphi(V), \varphi(E)) = G$



Symmetry Detection on Graphs

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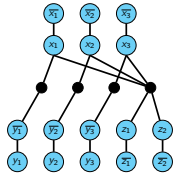

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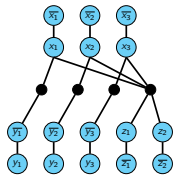

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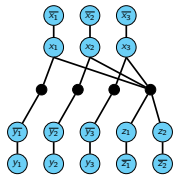

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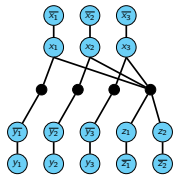

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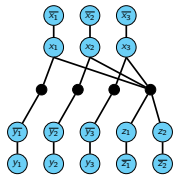

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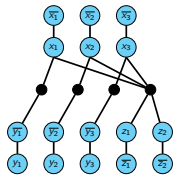

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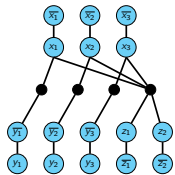

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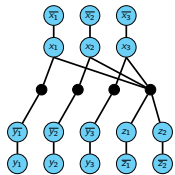

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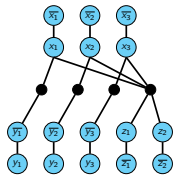

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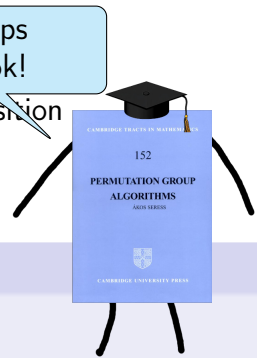

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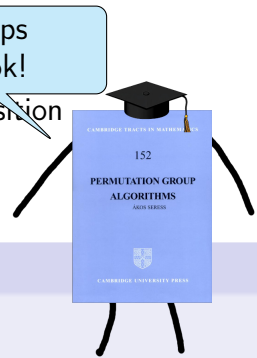
Output: *Generating set* $S \subseteq \mathrm{Aut}(G)$ *with* $\langle S \rangle = \mathrm{Aut}(G)$

Tread carefully...

- a graph G can have an exponential number
- symmetries $\mathrm{Aut}(G)$ form a permutation group under composition
- if φ and φ' are symmetries, so is $\varphi \circ \varphi'$
- only write down small subset S which *generates* $\mathrm{Aut}(G)$



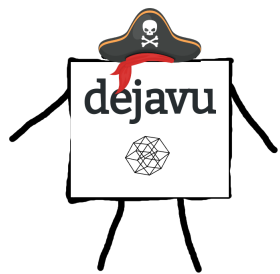
Problem (Symmetry Detection)

Input: *Graph* G
Output: *Generating set* $S \subseteq \mathrm{Aut}(G)$ *with* $\langle S \rangle = \mathrm{Aut}(G)$

- state-of-the-art tools are nauty, saucy, bliss, Traces, dejavu

Symmetry Detection: dejavu

- I've been building the symmetry detection tool `dejavu` for the past 5 years



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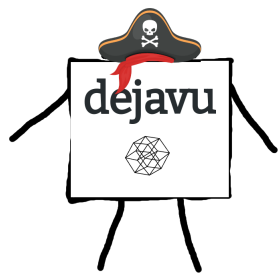

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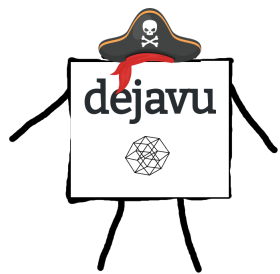

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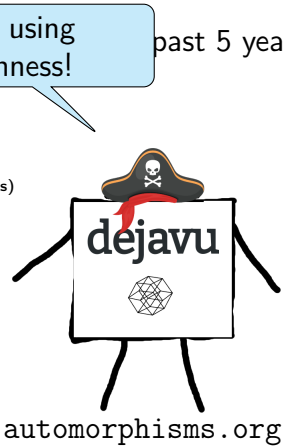

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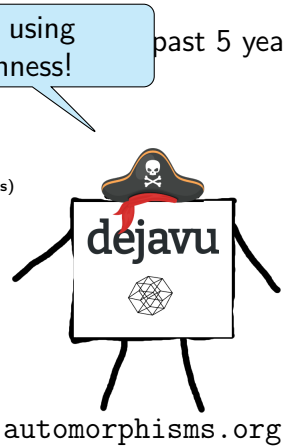
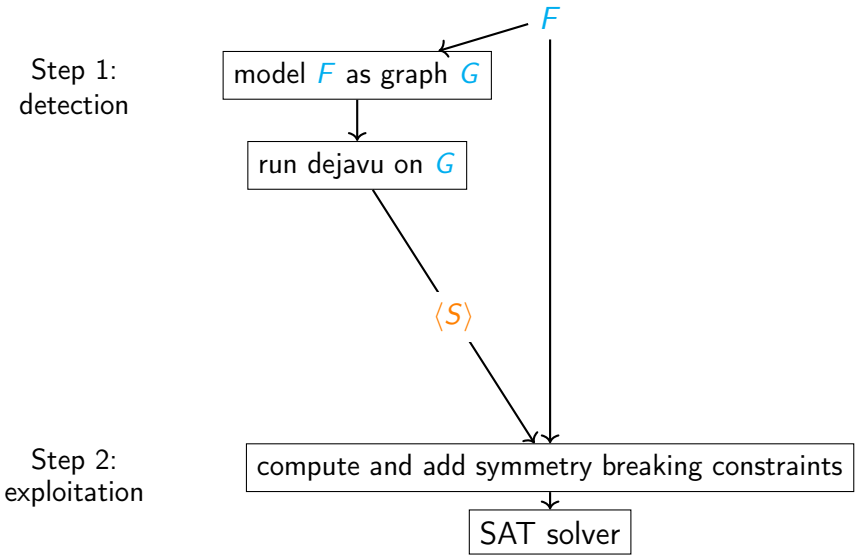
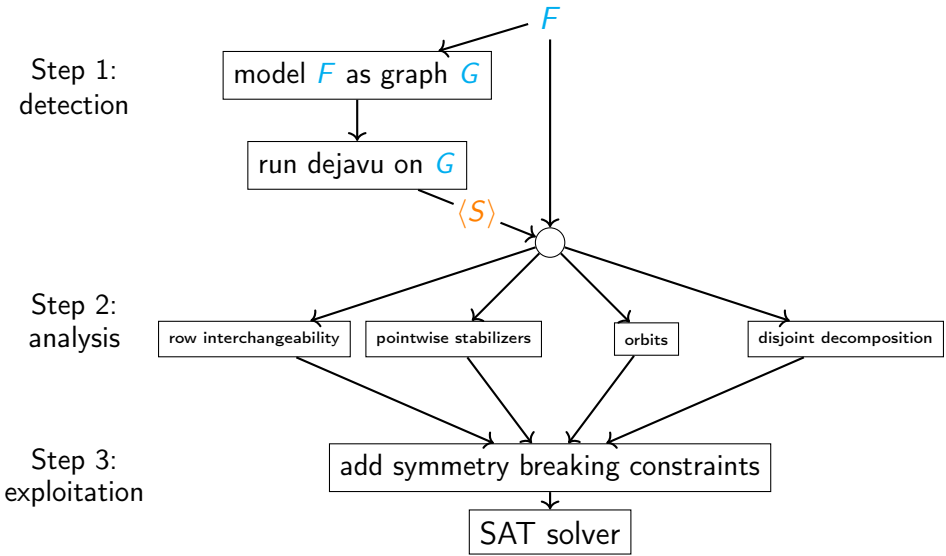
- randomness is *inherent* to the design



Symmetry Breaking: Refined Picture



Symmetry Breaking: Refined Picture II



Symmetry Breaking Sub-Tasks

row interchangeability | pointwise stabilizers | orbits | disjoint decomposition

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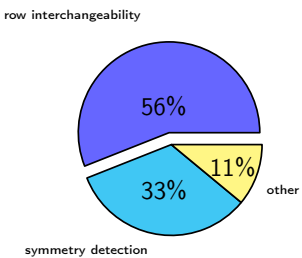

BreakID on PHP instances

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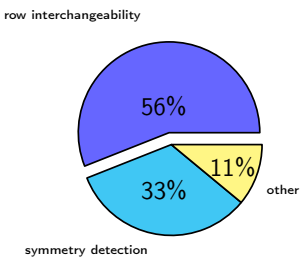

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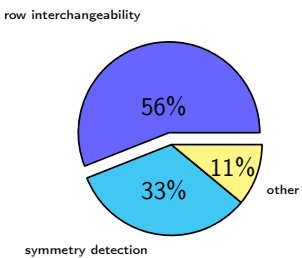

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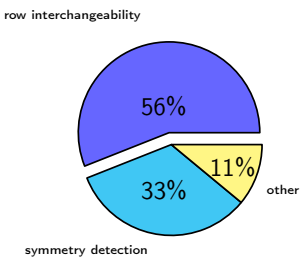

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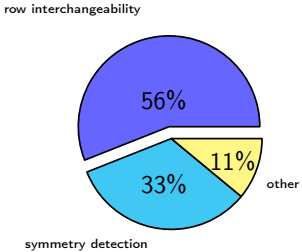

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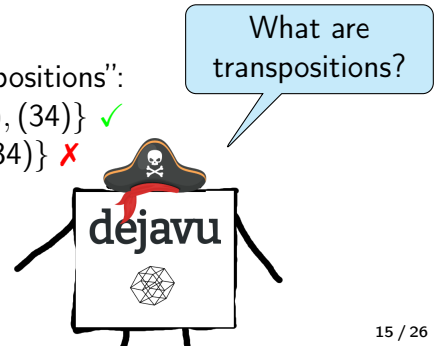

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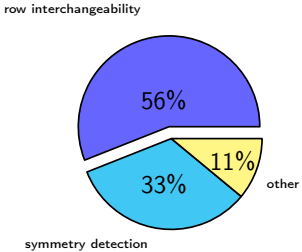

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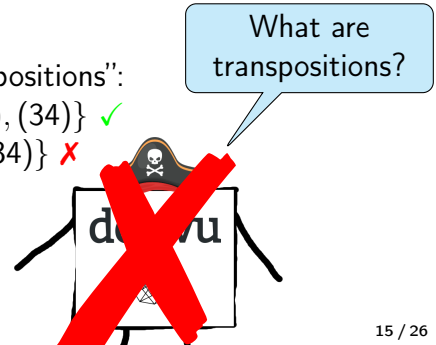

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- So... also not *quite* what we want?

SAT-Symmetry Algorithms

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Given a SAT formula F, graph $G = (V, E)$, we call algorithms that run in time $\mathcal{O}(|F| + |V| + |E| + \mathrm{enc}(S))$ *instance-linear*, where $\mathrm{enc}(S) := \Sigma_{p \in S} |\mathrm{supp}(p)|$.

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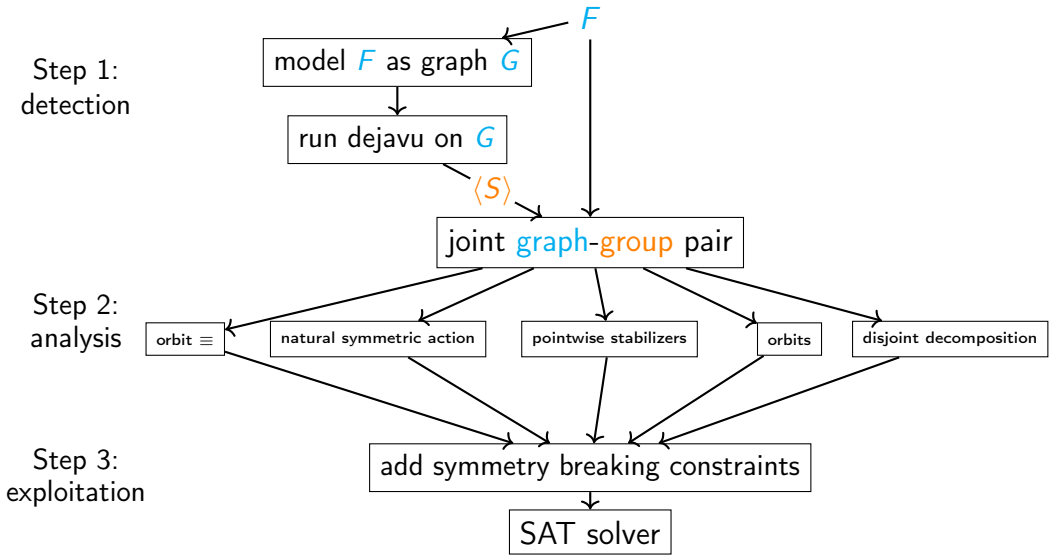
Symmetry Breaking: Refined Picture III



Algorithms in the Paper

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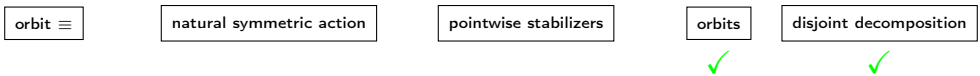

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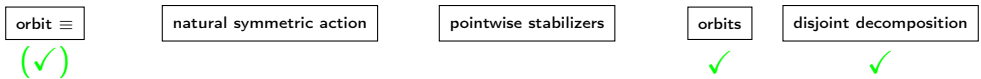

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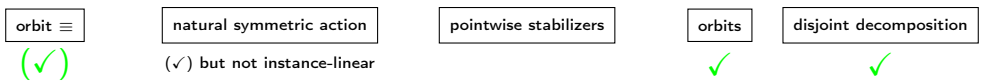

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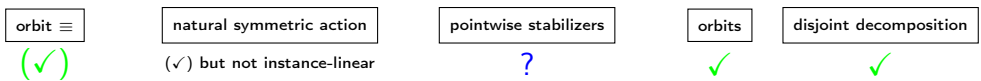

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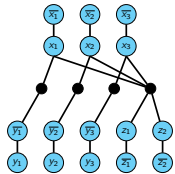
Finest Disjoint Direct Decomposition: What it's about



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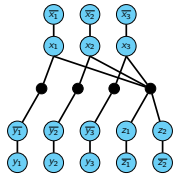

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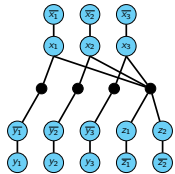

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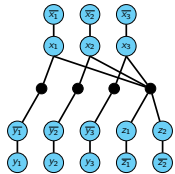

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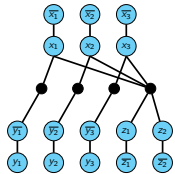

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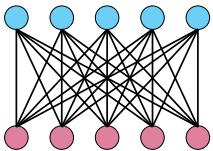
Here they are not?

Finest Disjoint Direct Decomposition: Flip edges



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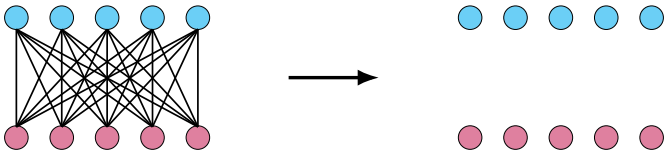

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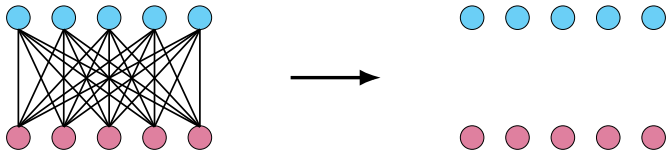

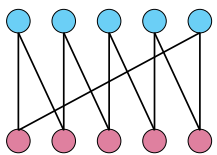

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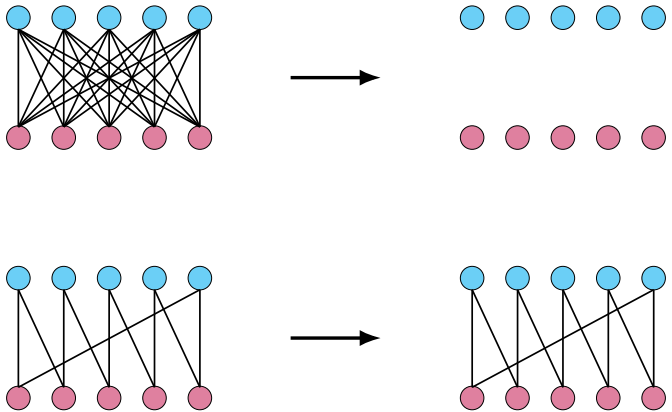

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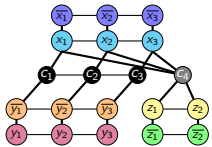

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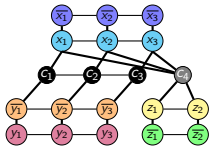
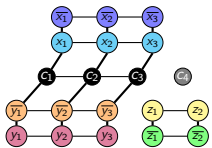


2. flip edges between colors



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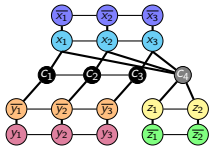

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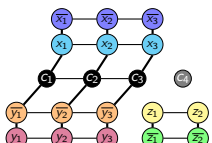

3. compute connected components

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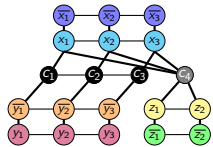

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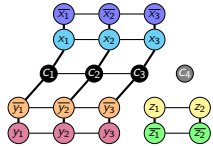

3. compute connected components

Lemma

Vertices are in the same connected component if and only if they are in the same factor of the finest direct disjoint decomposition.

Finest Disjoint Direct Decomposition: Algorithm

- we can split generators according to connected components

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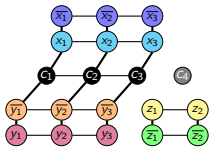

Finest Disjoint Direct Decomposition: Algorithm

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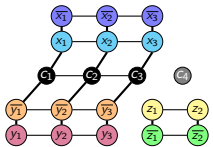

$S = \{$
$(x_1x_2x_3)(\overline{x_1x_2x_3})(y_1y_2y_3)(\overline{y_1y_2y_3}),$
$(x_1x_2)(\overline{x_1x_2})(y_1y_2)(\overline{y_1y_2})(z_1z_2)(\overline{z_1z_2}),$
$(z_1z_2)(\overline{z_1z_2})\}$

not independent

Finest Disjoint Direct Decomposition: Algorithm

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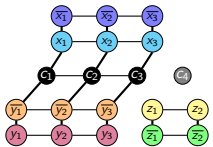

$S = \{$
$(x_1x_2x_3)(\overline{x_1x_2x_3})(y_1y_2y_3)(\overline{y_1y_2y_3}),$
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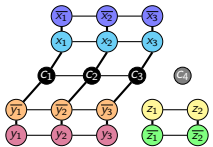

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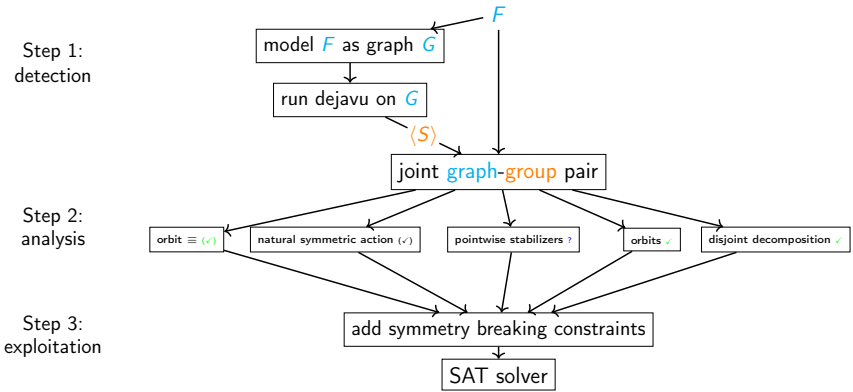
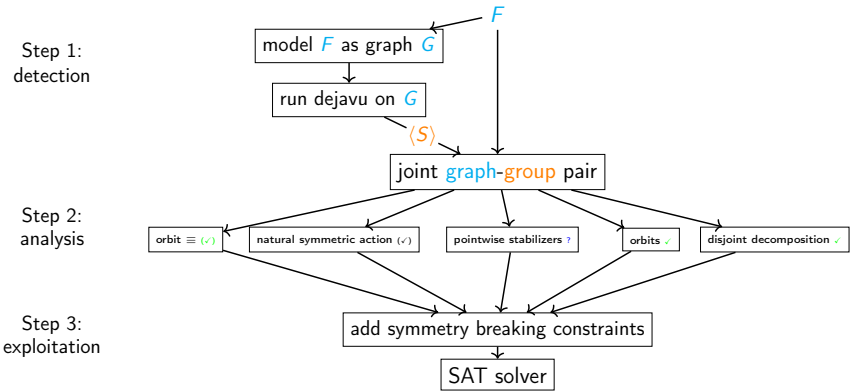
independent

Summary

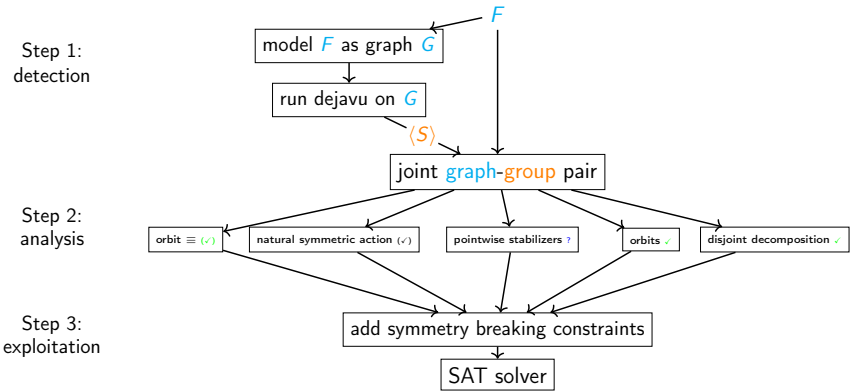

Step 1:
detection
Step 2:
analysis
Step 3:
exploitation
F
model F as graph G
run dejavu on G
⟨S⟩
joint graph-group pair
orbit ≡ (✓)
natural symmetric action (✓)
pointwise stabilizers ?
orbits ✓
disjoint decomposition ✓
add symmetry breaking constraints
SAT solver

Summary



- make use of joint graph-group pairs to get fast, generic algorithms and heuristics

Summary



- make use of joint graph-group pairs to get fast, generic algorithms and heuristics
- work to do to put this in practice (no 1:1 replacement for heuristics!)

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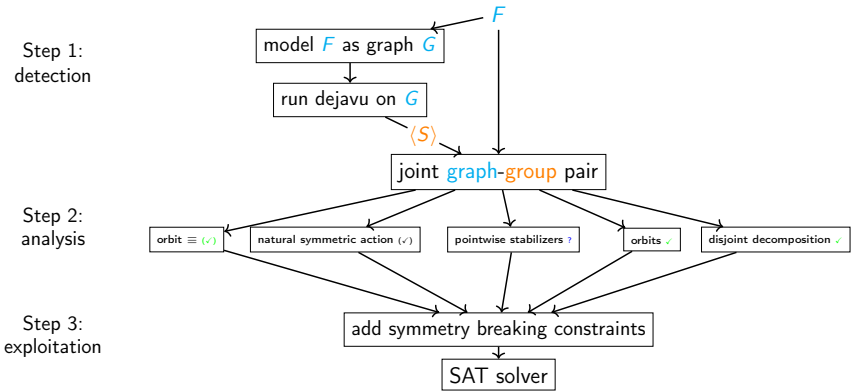

- make use of joint graph-group pairs to get fast, generic algorithms and heuristics
- work to do to put this in practice (no 1:1 replacement for heuristics!)
- want to detect more involved group structures